Coupling of brane multiplets in 5-D supergravity orbifolds

B. C. Georgalas, P. Kouroumalou , A. B. Lahanas,& G.A.D.

University of Athens, Physics Department, Nuclear and Particle Physics Section

- Introduction
- 5-D Supergravity Orbifold
- Coupling of Brane multiplets
- Discussion - Conclusions

Introduction

- Extra Dimensions
- Cosmological Implications
- Phenomenological Implications

Supersymmetry Breaking transmitted through the bulk.

Mirabelli, Peskin.

Gravity mediated Supersymmetry Breaking.

Gherghetta, Riotto, Rattazzi, Scrucca, Strumia.

GUT symmetry breaking as a bulk effect.

Georgalas, Kouroumalou, Lahanas, D.

⋆ **Off-shell formulation**

Kugo, Ohashi, Zucker, Gherghetta, Riotto, Rattazzi, Scrucca, Strumia

⋆ **On-shell formulation**

Gunaydin, Siera, Townsend, Falkowski, Lalak, Pokorski

5-D Supergravity Orbifold

- The supergravity multiplet.

$$\{e_{\tilde{\mu}}^{\tilde{m}}, \Psi_{\tilde{\mu}}^{i}, A_{\tilde{\mu}}\}$$

fünfbein $e_{\tilde{\mu}}^{\tilde{m}}$, two gravitini $\Psi_{\tilde{\mu}}^{i}$, graviphoton $A_{\tilde{\mu}}$

$\tilde{\mu} = (\mu, 5)$ are curved and $\tilde{m} = (m, \dot{5})$ are flat five-dimensional indices, with μ, m their four dimensional part and $i = 1, 2$ is the symplectic $SU(2)_R$ index.

On-shell Lagrangian

$$\begin{aligned}
\mathcal{L}_0 / e^{(5)} &= -\frac{1}{2} R^{(5)} + \frac{i}{2} \bar{\Psi}_{i\tilde{\mu}} \gamma^{\tilde{\mu}\tilde{\nu}\tilde{\rho}} \nabla_{\tilde{\nu}} \Psi_{\tilde{\rho}}^{i} - \frac{1}{4} F_{\tilde{\mu}\tilde{\nu}} F^{\tilde{\mu}\tilde{\nu}} \\
&+ \frac{3i}{8\sqrt{6}} \left[\bar{\Psi}_{i\tilde{\mu}} \gamma^{\tilde{\mu}} \gamma^{\tilde{\nu}} \gamma^{\tilde{\rho}} \gamma^{\tilde{\sigma}} \Psi_{\tilde{\nu}}^{i} F_{\tilde{\rho}\tilde{\sigma}} + 2 \bar{\Psi}_{i\tilde{\mu}} \Psi_{\tilde{\nu}}^{i} F^{\tilde{\mu}\tilde{\nu}} \right] \\
&+ \frac{\epsilon^{\tilde{\mu}\tilde{\nu}\tilde{\rho}\tilde{\sigma}\tilde{\lambda}}}{6\sqrt{6} e^{(5)}} F_{\tilde{\mu}\tilde{\nu}} F_{\tilde{\rho}\tilde{\sigma}} A_{\tilde{\lambda}} + (4 - \text{fermion})
\end{aligned}$$

- Supersymmetry transformations

$$
\begin{aligned}
\delta e_{\tilde{\mu}}^{\tilde{m}} &= i\bar{\epsilon}_i \gamma^{\tilde{m}} \Psi_{\tilde{\mu}}^i \\
\delta \Psi_{\tilde{\mu}}^i &= 2\nabla_{\tilde{\mu}}(\hat{\omega})\epsilon^i - \frac{1}{2\sqrt{6}} \left(\gamma_{\tilde{\mu}}^{\tilde{\nu}\tilde{\rho}} + 4\delta_{\tilde{\mu}}^{\tilde{\nu}} \gamma^{\tilde{\rho}} \right) F_{\tilde{\nu}\tilde{\rho}} \epsilon^i \\
\delta A_{\tilde{\mu}} &= -\frac{i\sqrt{6}}{2} \bar{\Psi}_{\tilde{\mu} i} \epsilon^i
\end{aligned}
$$

- S_1/Z_2 orbifold .

The 5th dimension is considered to be compact (circle) and the S_1/Z_2 orbifold is conceived by assigning appropriate Z_2 parity to the fields.

Z_2-even fields	e_μ^m, $e_5^{\dot{5}}$, Ψ_μ^1, Ψ_5^2, A_5
Z_2-odd fields	$e_\mu^{\dot{5}}$, e_5^m, Ψ_μ^2, Ψ_5^1, A_μ

The parity assignments are consistent with the the supersymmetry transformations if the corresponding parameters

$$\epsilon^1 = \begin{pmatrix} \varepsilon \\ \bar{\zeta} \end{pmatrix}, \; \epsilon^2 = \begin{pmatrix} \zeta \\ -\bar{\varepsilon} \end{pmatrix}$$

are taken to consist of even ε and odd ζ.

4-D supergravity multiplet $\Longrightarrow$ $\{ \, e_\mu^m, \psi_\mu \equiv \; \Psi_\mu^1 \}$

Radion multiplet $\Longrightarrow$ $\{ \, T \equiv \frac{1}{\sqrt{2}} e_5^{\dot{5}} + i\sqrt{\frac{1}{3}} A_5, \;\; \Psi_5^2 \}$

Coupling of Brane multiplets

Chiral multiplets live on the branes at $x^5 = 0\,, \;\; x^5 = \pi R$. Assume a globally supersymmetric $\sigma-$model and seek for the coupling of this model to the bulk fields in order to have invariance under local $N = 1$ supersymmetry transformations parametrized by the parameter ε. For one chiral multiplet at the brane $x^5 = 0$ the brane action is:

$$
\begin{aligned}
\mathcal{L}_b &= -e^{(5)} \Delta_{(5)} \Big[K_{\varphi\varphi^*} D_\mu \varphi D^\mu \varphi^* \\
&+ \left(\frac{i}{2} K_{\varphi\varphi^*} \chi \sigma^\mu D_\mu \bar{\chi} \,+\, h.c\right) \\
&+ \frac{1}{2} \left(D_\varphi D_\varphi W \, \chi\chi \,+\, h.c.\right) \\
&+ K^{\varphi\varphi^*} D_\varphi W \; D_{\varphi^*} W^* \\
&- \frac{1}{4} R_{\varphi\varphi^*\varphi\varphi^*} \chi\chi\bar{\chi}\bar{\chi} \Big] \,, \qquad \Delta_{(5)} = e^{\dot{5}}_{5} \delta(x^5)
\end{aligned}
$$

a. Ignore the radion multiplet.

If we ignore the radion multiplet the Nöther method for the coupling of the brane fields to the vierbein and the gravitino proceeds almost just as in the four-dimensionsl case. In terms of the $U_A(1)$ currents

$$J_\mu^{(\varphi)} = -i\,(K_\varphi \partial_\mu \varphi - K_{\varphi^*} \partial_\mu \varphi^*) \qquad J_\mu^{(\chi)} = K_{\varphi\varphi^*} \chi \sigma_\mu \bar{\chi}.$$

the resulting Lagrangian is :

$$
\begin{aligned}
\mathcal{L}^{(4)} = e^{(5)} \Delta_{(5)} \Big[& - K_{\varphi\varphi^*} D_\mu \varphi D^\mu \varphi^* \\
& - \left(\frac{i}{2} K_{\varphi\varphi^*} \chi \sigma^\mu D_\mu \bar{\chi} + h.c \right) \\
& - \frac{1}{\sqrt{2}} K_{\varphi\varphi^*} \left(D_\mu \varphi^* \chi^i \sigma^\nu \bar{\sigma}^\mu \psi_\nu + D_\mu \varphi \bar{\chi} \bar{\sigma}^\nu \sigma^\mu \bar{\psi}_\nu \right) \\
& + \frac{i}{4} E^{\mu\nu\rho\sigma} \left(J_\sigma^{(\chi)} - J_\sigma^{(\varphi)} \right) \psi_\mu \sigma_\nu \bar{\psi}_\rho \\
& + \frac{1}{4} J_\sigma^{(\chi)} \psi_\mu \sigma^\sigma \bar{\psi}^\mu + \frac{1}{4} \Delta_{(5)} \, J^{(\chi)\,\mu} \left(J_\mu^{(\varphi)} + \frac{1}{4} J_\mu^{(\chi)} \right) \\
& - \frac{1}{8} R_{\varphi\varphi^*\varphi\varphi^*} \, \chi \sigma^\mu \bar{\chi} \, \chi \sigma_\mu \bar{\chi} \\
& - e^{\Delta_{(5)} \, K/2} \left(W^* \psi_\mu \sigma^{\mu\nu} \psi_\nu + \frac{i}{\sqrt{2}} D_i W \chi^i \sigma^\mu \bar{\psi}_\mu \right. \\
& \left. + \frac{1}{2} D_i D_j W \chi^i \chi^j + h.c. \right) \\
& - e^{\Delta_{(5)} \, K} \left(K^{\varphi\varphi^*} \, D_\varphi W \, D_{\varphi^*} W^* - 3 \Delta_{(5)} \, |W|^2 \right) \Big]
\end{aligned}
$$

b. Coupling with the radion multiplet.

Proceeding with the Nöether method to derive the coupling of the brane fields with the radion multiplet completing the spectrum of the even fields of 5-D Supergravity one finds new terms of the form:

$$(J^{(\varphi)\,\mu} - \frac{1}{2} J^{(\chi)\,\mu})\; F_{\mu\dot{5}}\;, \quad K_{\varphi\varphi^*}\; D_\mu \varphi^* \chi \sigma^\mu \bar{\psi}^2_5\;, \quad K_\varphi\; \chi \sigma^\mu D_\mu \bar{\psi}^2_5$$

and besides these terms φ-dependent contributions to the kinetic terms of the radion multiplet fields appear, of the form:

$$K\; D_\mu e^{\dot{5}}_5 D^\mu e^{\dot{5}}_5\;, \quad K\; F_{\mu\dot{5}} F^{\mu\dot{5}}\;, \quad K \psi^2_5 \sigma^\mu D_\mu \bar{\psi}^2_5\;.$$

This procedure turns out to be very complicated so we'd like to find an alternative easier way to derive these terms using standard knowledge of 4-D Supergravity.

The restriction of 5-D Supergravity on the brane is invariant under ε transformations so it has to coinside with the Lagrangian of 4-D Supergravity coupled to a chiral multiplet, the radion multiplet. Writing this restriction we make the following remarks.

i). The pure supergravity part is not of the Einstein form but is multiplied by the radion field $e_5^{\dot{5}} = \frac{1}{\sqrt{2}}(T + T^*)$.

ii). There is no kinetic term for $e_5^{\dot{5}}$ and the spinor, ψ_5^2.

Look if the above properties emerge from a typical 4-D Supergravity.

- Assume a "frame" which we call 4-D Einstein frame where the pure Supergravity part is just of the Einstein form and the coupling of the radion multiplet is described by a "Kähler " function $\Lambda(T,T^*)$.
- Try to determine the "Kähler " function and the transformations in order to derive the restriction of the 5-D Supergravity on the brane.

Indeed if we take:

$$\Lambda(T,T^*) = -3ln\left(\frac{T+T^*}{\sqrt{2}}\right),$$

a Weyl rescaling

$$e_\mu^m \to e^f e_\mu^m \;, \quad \psi_\mu \to e^{f/2}\psi_\mu \;, \quad \psi_5^2 \to e^{-f/2}\psi_5^2$$

where

$$e^{2f} = e_5^{\dot{5}} = \frac{T + T^*}{\sqrt{2}}$$

followed by a shift of the gravitino field:

$$\psi_\mu \to \psi_\mu - \frac{i}{2}\sigma_\mu \bar{\psi}_5^2 e^{-2f}$$

produces the restriction of 5-D Supergravity on the brane.

- To incorporate the brane multiplet $(\varphi\,,\chi)$ we assume that in the 4-D Einstein frame the Lagrangian is determined by a "Kähler " function

$$\mathcal{F} = \Lambda(T,T^*) + \mathcal{K}(T,T^*,\varphi,\varphi^*)$$

where $\mathcal{K}$ describes the brane-radion multiplet interaction.

- Consider the kinetic term of the scalar field φ in this frame. The Weyl rescaling results to

$$e^{(4)} e_5^{\dot{5}} \mathcal{K}_{\varphi\varphi^*} \partial_\mu \varphi \partial^\mu \varphi^*,$$

- which is the scalar kinetic term we started with if we set

$$\mathcal{K} \;=\; \Delta_{(5)} K \;=\; \delta(x^5) \frac{\sqrt{2}}{T+T^*} K.$$

All the new terms that appeared applying the Nöether method are produced by this procedure.

c. Potential.

- Start with the potential terms in $\mathcal{L}^{(4)}$

$$e^{(5)}\Delta_{(5)}e^{\Delta_{(5)}\ K}\big(\ \frac{1}{K_{\varphi\varphi^*}}\ D_\varphi W\ D_{\varphi^*}W^* - 3\Delta_{(5)}\ |W|^2\ \big)$$

- Weyl rescale to see the form in the 4-D Einstein frame. The result is

$$e^{(5)}\Delta_{(5)}e^{\mathcal{F}}\big(\ \frac{1}{e^{-2f}K_{\varphi\varphi^*}}\ D_\varphi W\ D_{\varphi^*}W^* - 3\delta(x^5)\ |W|^2\ \big).$$

Note the exponentiation of the "Kähler " function $\mathcal{F}$ in this frame.

- **The potential in the 4-D Einstein frame, including the coupling to the radion multiplet reads:**

$$e^{(5)}\Delta_{(5)}e^{\mathcal{F}}\left(\mathcal{F}^{j^*i}\ D_iW\ D_{j^*}W^* - 3\delta(x^5)\ |W|^2\ \right).$$

- **This is the ordinary 4-D Supergravity potential with a ”Kähler ” function $\mathcal{F}$ with the exception of the extra $\delta(x^5)$.**
- Note that metric becomes two-dimensional even in the case of just one brane multiplet, ($i, j = T, \varphi$).
- The inverse metric $\mathcal{F}^{ij^*}$ is defined by

$$\mathcal{F}_{ij^*} \, \mathcal{F}^{j^* k} \; = \; \delta_i^k \delta(x^5).$$

This normalization is natural since $\mathcal{F}$ has dimension 3 while the inverse metric has to be dimensionless.

- **In the 5-D Einstein frame the potential is:**

$$e^{(5)} \Delta_{(5)} e^{\Delta_{(5)} \, K} \left(e_{\dot{5}}^{5} \mathcal{F}^{j^* i} \, D_i W \, D_{j^*} W^* - 3\Delta(x^5) \, |W|^2 \right).$$

Discussion - Conclusions

- Singular terms $\sim \delta^2(x^5)$.

Important at the quantum level.

Non-singular classical 4-dimensional effective action is conjectured in literature.

$W = 0$.

The appearence of the perfect square

$$\left[F_{\mu\dot{5}} - \frac{1}{\sqrt{6}} \left(J_\mu^{(\varphi)} - \frac{1}{2} J_\mu^{(\chi)} \right) \right]^2$$

is crucial for the absence of the singularity.

This occurs in a particular frame the **Weyl frame**:

$$e_\mu^m = \omega\, \tilde{e}_\mu^m , \;\; \omega^2 = -\frac{1}{3}\Omega \, , \qquad \Delta_{(5)}\, K = -3\, ln(-\frac{\Omega}{3}) \; .$$

Performing consistently the Weyl rescaling we find a perfect square of the matter currents which combined with the terms that are linear and bilinear in $F_{\mu\dot{5}}$ yields a perfect square given by

$$\tilde{e}^{(5)}\, \frac{3}{2\Omega} \left[\hat{F}_{\mu\dot{5}} - \frac{1}{\sqrt{6}} \left(\hat{J}_\mu^{(\varphi)} - \frac{1}{2} \hat{J}_\mu^{(\chi)} \right) \right]^2$$

where $\hat{F}_{\mu\dot{5}} \equiv (-\Omega/3) F_{\mu 5}\; e_{\dot{5}}^{5}$.
(Rattazzi, Scrucca, Srtumia)

⇒ Nonsingular classical action up to k_5^2 for the brane fields.

$W \neq 0$.

New singular terms from the exponentiation of the "Kähler" function and from the $|W|^2$ term of the potential. This last term is unpleasent since

i). It cannot be cancelled by tree level graphs with bulk field exchange (Mirabelli, Peskin).

ii). In all frames discussed so far it remains unchanged (up to factors).

Only the coupling with the radion multiplet can cure this singularity.

Indeed if we consider

$$e_{\dot{5}}^{5} \mathcal{F}^{TT^*} \; D_T W \; D_{T^*} W^*$$

and expand it in powers of k_5 for any $K \; = \; \varphi\varphi^* + \mathcal{O}(k)$, we get a term

$$3\Delta_{(5)} \; |W|^2$$

cancelling that in the potential.

$\Rightarrow$ In the "Weyl" frame **all** the singular terms in the potential cancel up to k_5^2.

- The coupling of a brane multiplet to 5-D Supergravity to include the interactions with the radion multiplet is obtained as follows:

1. Add to $\mathcal{L}^{(4)}$ all the terms mixing the brane fields and the radion multiplet, dictated by the improved "Kähler " function $\mathcal{F}$
2. For the potential use $\mathcal{F}$ to derive the metric $\mathcal{F}_{j^* k}$ and to covariantize the derivatives. e.g.

$$\frac{1}{K_{\varphi\varphi^*}} D_\varphi W \; D_{\varphi^*} W^* \longrightarrow e^{\dot{5}}_{5} \mathcal{F}^{j^* i} \; D_i W \; D_{j^*} W^*$$

3. Shift the gravitino.

- ∂_5 of odd fields are included by $\partial_\mu A_5 \to F_{\mu 5}$, $D_\mu \psi^2_5 \to D_\mu \psi^2_5 - D_5 \psi^2_\mu$.

- The above procedure can be applied to include vector multiplets on the brane.
- Alternatively starting from 5-D Yang-Mills Supergravity one can allow for gauge

interactions propagating in the bulk.

- Using of Gauged 5-D Supergravity bulk potential can also be incorporated.
- Supersymmetry Breaking

Potential on the hidden brane at $x^5 = \pi R$.

Radion field expectation value.

- Transmission through the bulk.

Calculable one-loop corrections to the brane field masses.
Gravity mediated
Radion mediated

Anomaly mediated

Work in progress $\cdots$