Neutrino Masses From Flavor Symmetries

G.K. Leontaris, J. Rizos and A. Psallidas

University of Ioannina
Section of Theoretical Physics
GR-451 10 Ioannina, Greece

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Neutrino Oscillation Data

- Masses
  - $\Delta m^2_{\odot} \sim 10^{-5} eV^2$
  - $\Delta m^2_A \sim 10^{-3} eV^2$
- Mixing $(\sin\theta_{ij} \to s_{ij}\ ,\ \cos\theta_{ij} \to c_{ij})$

$$U_\ell = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta} & c_{13}s_{23}e^{-i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{-i\delta} & c_{13}c_{23}e^{-i\delta} \end{pmatrix}$$

$$\theta_{23} \approx \frac{\pi}{4} \quad , \ \theta_{12} \approx \frac{\pi}{5.4} \quad , \ \theta_{13} \approx 0$$

Quark-Lepton Complementarity

Mixing matrix $\longrightarrow$ Close to BIMAXIMAL

$$\text{QLC :} \quad \theta_{12} + \theta_C = \frac{\pi}{4} \tag{1}$$

where θ_C: Cabibbo angle who in the Wolfenstein parametrization show us the deviation of the quark mixing matrix from identity.

Suggestion of a deeper structure by which quark and leptons are interrelated $\longrightarrow$ Quark-Lepton Symmetry

Sources Of Neutrino Mass

- $\frac{1}{M}(\bar{L}^c H)(LH) \longrightarrow m_\nu \, \bar{\nu}_L^c \nu_L$ 5 dimensional operator
- Dirac term $\longrightarrow$ $m_D \, \nu_R^c \nu_L$ 4 d.f with the same mass
- Heavy Majorana $\longrightarrow$ $M_R \, \nu_R^c \nu_R$ 2 d.f with the same mass

After See-Saw Mechanism

$$m_\nu^{\text{eff}} = m_\nu - \frac{1}{4} m_D M_R^{-1} m_D^T \quad (2)$$

We explore the case

$$m_\nu^{\text{eff}} = m_\nu \quad (\text{absence of } \nu_R) \quad (3)$$

Motivation

- Models of Large Extra Dimensions
  - Absence of M_R
  - Intermediate scaled Gravity $(10^{11-13}\ GeV)$
  - Prediction of Extra U(1)'s
  - Minimal model ("economy")

A Simple Model

- MSSM is considered as low energy effective theory of a string or a brane model.

  Remnants of the higher theory:

- $U(1)$ symmetry (possibly anomalous)

$$G_{SM} \longrightarrow G_{SM} \times U(1)_X$$

- Singlet Higgs Fields $\phi, \bar{\phi}\ \ldots \quad (\langle\phi\rangle\ ,\ \langle\bar{\phi}\rangle) \neq 0$
  - ☛ All Standard Model Fields are ”charged” under the extra $U(1)$.

Fields

Fermion	Higgs
$Q_i(3,2,\frac{1}{6};q_i)$	$H_1(1,2,-\frac{1}{2};h_1)$
$D_i(\bar{3},1,\frac{1}{3};d_i)$	
	$H_2(1,2,\frac{1}{2};h_2)$
$U_i(\bar{3},1,-\frac{2}{3};u_i)$	
$L_i(1,2,-\frac{1}{2};\ell_i)$	$\Phi(1,1,0;+1)$
$E_i(1,1,1;e_i)$	$\bar{\Phi}(1,1,0;-1)$

Consequences of Extra $U(1)_X$

- Only terms invariant under $G_{SM} \times U(1)_X$ are present in the superpotential.

Assumptions

- Hierarchy $\longrightarrow$ only 3^{rd}-family mass couplings at tree-level

$$\mathcal{W}_{tree} \sim \quad Q_3 U_3^c H_2 \ , \ Q_3 D_3^c H_1 \ , \ L_3 E_3^c H_1 \qquad (4)$$

$$\begin{aligned} q_3 + u_3 + h_2 &= 0 \\ q_3 + d_3 + h_1 &= 0 \\ \ell_3 + e_3 + h_1 &= 0. \end{aligned} \qquad (5)$$

☞ Symmetric mass matrices

$$Q_i U_j^c H_2 \left(\frac{\langle\bar{\phi}\rangle}{M}\right)^{r_{ij}} \quad \text{with} \quad q_i + u_j + h_2 - r_{ij} = 0 \tag{6}$$

$$\begin{aligned} C_{ij}^u &\equiv q_i + u_j = q_j + u_i \\ C_{ij}^d &\equiv q_i + d_j = q_j + d_i \\ C_{ij}^e &= \ell_i + e_j = \ell_j + e_i. \end{aligned} \tag{7}$$

☞ C_{ij}^u corresponds to the $U(1)_X$-charge of the (ij)-quark mass entry.

Quark and Lepton $U(1)_X$-charge look like:

$$C^u = C^d = \begin{pmatrix} n & \frac{m+n}{2} & \frac{n}{2} \\ \frac{m+n}{2} & m & \frac{m}{2} \\ \frac{n}{2} & \frac{m}{2} & 0 \end{pmatrix} \tag{8}$$

$$q_1 - q_3 = \frac{n}{2}\,,\; q_2 - q_3 = \frac{m}{2} \;\text{ where } m+n \neq 0,\; m,n = \pm 1, \pm 2, \ldots$$

thus, quark matrices depend only on the two integers m, n!!!

- The zero entry is a consequence of the condition for the tree-level coupling.
- The remaining entries appear at appropriate Non-Renormalizable order.

$$Q_i U_j^c H_2 \varepsilon^{C_{ij}^u} \; , \; Q_i D_j^c H_1 \varepsilon^{C_{ij}^d} \; , \; L_i E_j^c H_1 \varepsilon^{C_{ij}^l} \tag{9}$$

where

$$\varepsilon^k = \begin{cases} \lambda^k & \text{if } k = [k] < 0 \\ \bar{\lambda}^k & \text{if } k = [k] > 0 \\ 0 & \text{if } k \neq [k] \end{cases} \tag{10}$$

Mass Matrices

- Quarks

$$M_{u,d} \sim m_0^{u,d} \begin{pmatrix} \lambda^n & \lambda^{\frac{m+n}{2}} & \lambda^{\frac{n}{2}} \\ \lambda^{\frac{m+n}{2}} & \lambda^m & \lambda^{\frac{m}{2}} \\ \lambda^{\frac{n}{2}} & \lambda^{\frac{m}{2}} & 1 \end{pmatrix} \quad (11)$$

- Charged Leptons

Charged leptons matrix is parametrised by two new parameters m', n'

$$C_\ell = \begin{pmatrix} n' & \frac{m'+n'}{2} & \frac{n'}{2} \\ \frac{m'+n'}{2} & m' & \frac{m'}{2} \\ \frac{n'}{2} & \frac{m'}{2} & 0 \end{pmatrix} \quad (12)$$

with $2n' = l_1 - l_3$, $2m' = l_2 - l_3$

- Neutrinos
  - Dimension-5 operator retained by $G_{SM} \times U(1)_X$

$$\frac{\zeta_\nu^{a\beta}}{M}(\bar{L_a^c}^i H^j \epsilon_{ji})(H^l L_\beta^k \epsilon_{lk})\bar{\lambda}^{n_{ij}}$$

$$C_\nu = \begin{pmatrix} n' + \mathcal{A} & \frac{m'+n'}{2} + \mathcal{A} & \frac{n'}{2} + \mathcal{A} \\ \frac{m'+n'}{2} + \mathcal{A} & m' + \mathcal{A} & \frac{m'}{2} + \mathcal{A} \\ \frac{n'}{2} + \mathcal{A} & \frac{m'}{2} + \mathcal{A} & \mathcal{A} \end{pmatrix} \quad (13)$$

where we have introduced the new parameter

$$\mathcal{A} = 2(l_3 + h_2) \quad (14)$$

- If A= integer $\Longrightarrow$ for positive charge entries (C_ν , C_ℓ) $\Longrightarrow$ charge lepton and neutrino matrices are ”proportional”:

$$m_\ell \approx \bar{\lambda}^A \; m_\nu$$

- LARGE MIXING NOT POSSIBLE

- A mass matrix entry is non-zero if the corresponding ”charge-entry ” is integer $\Longrightarrow \frac{m'}{2}$, $\frac{n'}{2}$ $=$ Integers or half-inegers

- Conditions for large mixing:

$$n' = \text{odd} \quad , \quad m' = \text{even} \quad , \quad 2\,A = \text{odd}$$

$$M_\ell = m_0^\ell \begin{pmatrix} \delta\,\lambda^{n'} & 0 & 0 \\ 0 & \lambda^{m'} & \alpha\,\lambda^{\frac{m'}{2}} \\ 0 & \alpha\,\lambda^{\frac{m'}{2}} & 1 \end{pmatrix} \tag{15}$$

$$M_\nu = m_0^\nu \begin{pmatrix} 0 & \zeta\,\lambda^{\frac{m'+n'}{2}+\mathcal{A}} & \lambda^{\frac{n'}{2}+\mathcal{A}} \\ \zeta\,\lambda^{\frac{m'+n'}{2}+\mathcal{A}} & 0 & 0 \\ \lambda^{\frac{n'}{2}+\mathcal{A}} & 0 & 0 \end{pmatrix} \tag{16}$$

Leptonic Mixing Matrix

$$U_l = V_l^\dagger V_\nu$$

$$U_l = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{\cos(\phi+\omega)}{\sqrt{2}} & -\frac{\cos(\phi+\omega)}{\sqrt{2}} & \sin(\phi+\omega) \\ \frac{\sin(\phi+\omega)}{\sqrt{2}} & \frac{\sin(\phi+\omega)}{\sqrt{2}} & \cos(\phi+\omega) \end{pmatrix}$$

$$\cot^2\omega = \zeta^2 \bar{\lambda}^{m'}$$

$$\tan\ 2\phi = \frac{2\alpha\bar{\lambda}^{\frac{m'}{2}}}{1-\bar{\lambda}^{m'}}$$

for $\phi + \omega = \frac{\pi}{4} \Longrightarrow$ Bimaximal

Specific Example

Solution A			
field	generation		
	1	2	3
Q	4	2	0
D^c	2	0	-2
U^c	4	2	0
L	$\frac{9}{4}$	$-\frac{1}{4}$	$-\frac{5}{4}$
E^c	$\frac{11}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$
Higgs			
H_1	2	H_2	0
Singlets			
Φ	1	$\bar{\Phi}$	-1

Table 1: Example of $U(1)_X$ charges which lead to the neutrino mass matrix structure discussed.

Specific Example

$$M_{u,d} \sim m_0^{u,d} \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} . \qquad (17)$$

$$M_\ell \sim m_0^e \begin{pmatrix} \delta\,\lambda^7 & 0 & 0 \\ 0 & \lambda^2 & a\,\lambda \\ 0 & a\,\lambda & 1 \end{pmatrix} \qquad (18)$$

$$M_\nu \sim m_0^\nu \begin{pmatrix} 0 & -\lambda^2 & \zeta\,\lambda \\ -\lambda^2 & 0 & 0 \\ \zeta\,\lambda & 0 & 0 \end{pmatrix} . \qquad (19)$$

Possible Sources of Higher Corrections

- Additional singlet fields $\chi, \bar{\chi}$

$$M_\nu = m_0^\nu \begin{pmatrix} 2x & -\cos\,\bar{\omega} & \sin\,\bar{\omega} \\ -\cos\,\bar{\omega} & 0 & 2y \\ \sin\,\bar{\omega} & 2y & 0 \end{pmatrix} \tag{20}$$

where $\bar{\omega} = \omega + \phi$.

- Mixing (diagonalising and identifying)

$$\begin{aligned} \tan\theta_{23} &\approx \tan\bar{\omega} \\ \tan\theta_{13} &\approx 2y\cos(2\bar{\omega}) \\ \tan\theta_{12} &\approx 1 - (x + y\sin(2\bar{\omega})) \end{aligned} \tag{21}$$

Conclusions

- $G_{SM} \times U(1)_X$-symmetry with MSSM spectrum.
  - Hierarchical mass spectrum for Quarks and Charged Leptons
  - Maximal mixing in Neutrino Sector
  - Inverted mass hierarchy for ν_i's $|m_1| \sim |m_2| >> |m_3|$
  - Applicable in Intermediate Gravity Models